# STATISTICAL MODELING OF DAMS BASED ON ORDER SERVICE PROCESSES AND POTENTIAL EVENTS OF FAILURE

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**ABSTRACT**

The safety operation of dam structures is a subject of complex characteristics of the flow which requires a suitable decision to be taken. Here the problem is examined by modeling the statistical evaluation and physical research. In practice many important structures such as spillways, energy dissipaters, intakes, gates, overflow are examined in detail. However failure events have not been included yet in the engineering assessments due to lack of data or lack of adequate information. In this paper three depending on the time processes of failure are considered during the operation time of radial gates. The description and analysis of potential events of failure include the following time-varying statistical processes: non-aging processes as Poisson and Erlang systems and aging processes. The inter arrival time and duration of failure are highly variable and depend not only on specific operation of the gates but affect hydraulics, hydrological and environmental characteristics of the dam structures. Some important parameters are calculated and evaluated by the model prototype and the hydraulic similarity in a laboratory. As a result the scale coefficients for linearity, velocity, pressure, discharge, time and force are obtained. Statistical tests are applied to choose the best distribution function at a given number of failure events.

*Keywords:* dam structure, radial gate, failure, model prototype, similarity, order service, statistics, intake device, settling basin, aging service process, serviceability

## 1 INTRODUCTION

The reliability of the dam operation is affected by the structural, environmental and engineering characteristics comprising the entire system. For example, the failure and blockage of dams may result in flooding, combined overflow events, dam body damages, input or output damages, etc. Hydraulic performance and assessment were currently carried out by the team of the hydraulic laboratory at the Department of Hydraulics and Hydrology at Sofia University. The operation of the system was modeled by a physical facility using an appropriate geometric scale – Fig.1. The components of the structure, represented in Fig.1, consist of the following parts:

- spillway with 9 radial gates – section D-D
- down stream section with energy dissipater – section E-E
- intake device and settling basin, presented in Fig. 2

The represented hydraulic structure is one of the main components for additional water supply situated near the capital Algeries (Algeria) and it was carried out by Bulgarian engineers. A water discharge of 28 $m^3s^{-1}$ gets over the settling basin, pumping station and finally it is transferring in water supply dam Djemaa Aval.

## 2 THE MODEL PROTOTYPE AND ITS SIMILARITY

The physical modeling is performed according to Reech-Froud criteria (Carlie 1972).

$$Fr_p = Fr_m \tag{1}$$

where the indices $p, m$ denote the Froud numbers for the prototype and for the model and this number is defined as

$$Fr = \frac{v^2}{gh} = idem \tag{2}$$

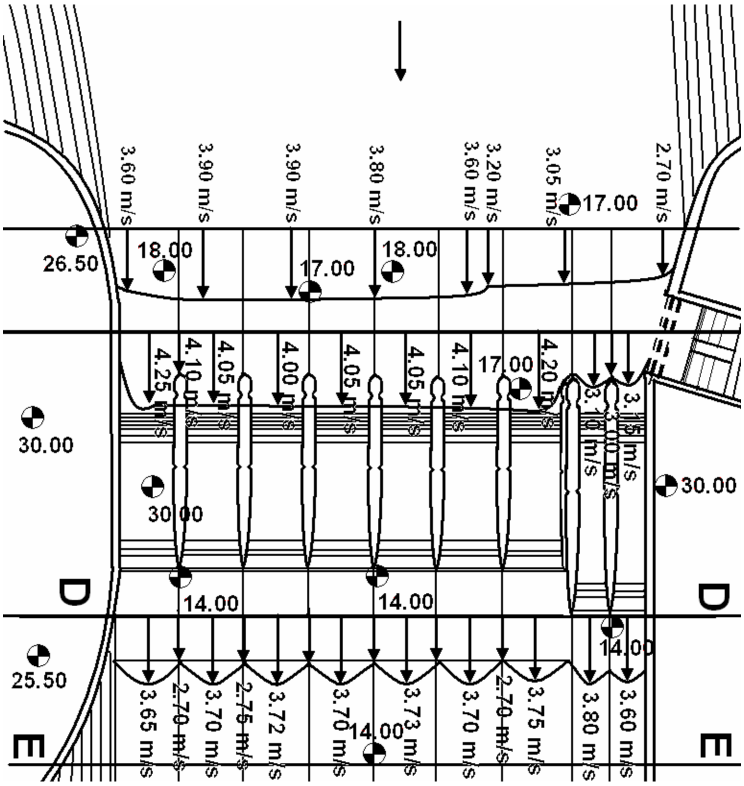


Fig.1 Hydraulic model of dam structure

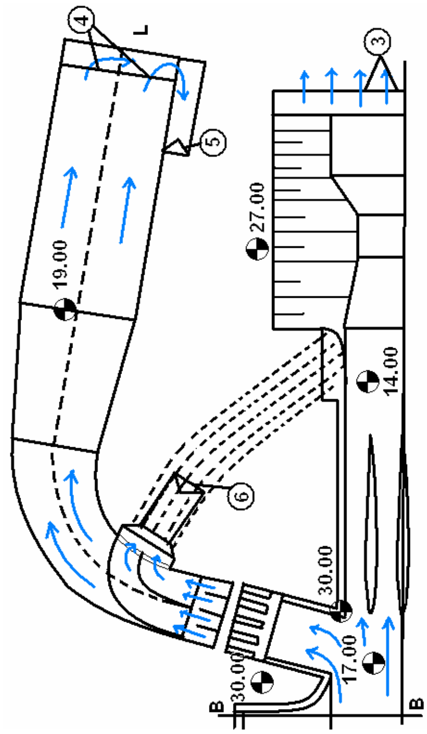


Fig.2 Model of intake device and settling basin

The probability water discharges in the prototype are $Q_{1\%}$, $Q_{0.1\%}$, $Q_{0.01\%}$ and this condition give the discharges 3530 $m^3s^{-1}$, 5000 $m^3s^{-1}$, 6400 $m^3s^{-1}$. The model flow must be turbulent and the turbulence level in the prototype should be the same as it is in the model. This condition is guaranteed by the following inequality:

$$\mathrm{Re}_m \geq \mathrm{Re}_{\lim} \tag{3}$$

where $\mathrm{Re}_{\lim}$ is the recommended limit value of Reynolds number.

This restriction gives the linear scale of modeling i.e.

$$M_l^{\min} = \left(\frac{\mathrm{Re}_p^r}{\mathrm{Re}_{\lim}}\right)^{2/3} \tag{4}$$

where the numerator is a representative Reynolds section for a prototype determined by a typical section.

Having in mind the representative section D-D, shown in Fig.1, the linear scale is $M_l^{\min} = 81$

The model scale was selected to be

$$M_l^{\min} = 75 \tag{5}$$

According to Manning the roughness coefficient of the model is

$$n_m = \frac{0.015}{75^{1/6}} = 0.0074 \approx 0.007 \tag{6}$$

The surface of the channel in section D-D was implemented with a smooth cement mixture and painted with latex, so that the coefficient of roughness for the model was approximately the same as that one in formula (6). Other scale coefficients are

$$
\begin{aligned}
&M_v = 8.66 && M_p = M_l^{\min} = 75 \\
&M_Q = \left(M_l^{\min}\right)^{2.5} && M_t = \left(M_l^{\min}\right)^{0.5} \\
&M_F = \left(M_l^{\min}\right)^{3} = 75^3 &&
\end{aligned}
\qquad (7)
$$

where $M_v$ is a scale for velocity, $M_p$ is a scale for pressure, $M_Q$, $M_t$, $M_F$ are scales for the discharge, time and force respectively.

## 3 STATISTICAL DESCRIPTION OF THE DAM SERVICEABILITY

Hydraulic models of dam structure indicate a trend in a time variation of upper water level. This fact affects the operation of radial gates and energy dissipater shown in Fig.1. The next figure shows the specific levels of the bottom, the normal level and crest configuration of the spillway.

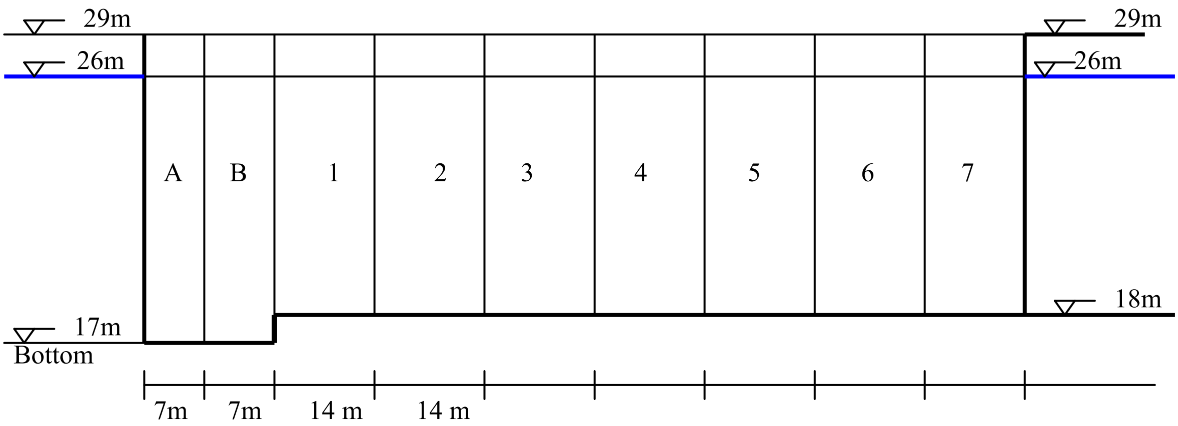


Fig.3 General Scheme of a spillway with 9 radial gates

The most common method for computation of the above mentioned structures includes preliminary specification of water discharges i.e. $Q_{1\%}$, $Q_{0.1\%}$, $Q_{0.01\%}$ so that the next calculation is completed correctly and the entire system should function successfully (Maradjieva M., Kazakov B., 2003). Having in mind the fact that the hydrological process is of statistical character the operation of hydraulic elements considerably depends on this random process. The description of potential events of failure includes the following time-varying statistical processes:

1. Poisson processes
2. Non Poisson processes
3. Aging services processes

These processes are discussed below but some elements that are a subject of the reliability theory are additional.

### 3.1 POISSON PROCESSES

These processes are of non aging character and satisfy the Poisson distribution in time. The time intervals between failures are submitted to the exponential distribution. As a rule from statistical point of view these processes obey to the Marcov chain (W. Lederman, 1989).

Many engineering applications are submitted to time depending probability events of failure known as Erlang flow. The density function, depending on the time t, is:

$$f_k(t) = \frac{\lambda(\lambda t)^k}{k!} e^{-\lambda t} \qquad t > 0 \tag{8}$$

where $\lambda, k$ are parameters.

The cumulative function is

$$F_k(t) = P(T < t) = 1 - \sum_{s=0}^{k} \frac{(\lambda t)^s}{s!} e^{-\lambda t} \tag{9}$$

The exponential distribution follows if $k = 0$ i.e. $f_0(t) = \lambda e^{-\lambda t} \qquad t > 0$.

By Poisson's formula

$$P(k, a) = \frac{a^k}{k!} e^{-a} \tag{10}$$

equations (8), (9) are transformed as

$$f_k(t) = \lambda P(k, \lambda t) \qquad a = \lambda t,\ t > 0 \tag{11}$$

$$F_k(t) = 1 - R(k, \lambda t) \tag{12}$$

$R(k, \lambda t) = \sum_{s=0}^{k} \frac{(\lambda\, t)^s}{s!} e^{-\lambda t}$ , $a = \lambda t, t > 0$ is the expected value or parameter in the Poisson formula.

The following relations exist on the basis of the above mentioned formulas

$$\frac{\partial}{\partial a} R(k, a) = -P(k, a)$$

$$\lim_{k \to \infty} R(k, a) = 1 \qquad \lim_{k \to \infty} P(k, a) = 0$$

Two different order service processes are known: with failure and with expectation. The first one is described with damages or with shortage of free lines. As a result some elements leave the system. The second case is possible and it can be reduced to Erlang equations. Both cases have the form of a recurrent algebraic system:

$$\begin{aligned}
&-\lambda P_0(t) + \mu P_1(t) = 0 \\
&-(\lambda + \mu) P_1(t) + \lambda P_0(t) + 2\mu P_2(t) = 0 \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
&-(\lambda + k\ \mu) P_k(t) + \lambda P_{k-1}(t) + (k+1)\mu P_{k+1}(t) = 0 \\
&-n\mu P_n(t) + \lambda P_{n-1}(t) = 0
\end{aligned} \tag{13}$$

Unknown probabilities are:

$P_k(t)$ is the probability given at the moment $t$ the state of the system is $x_k$ $(0 \le k \le n)$ and $k$ lines are damaged or occupied.

The parameters of the system are:

$n$ is the number of lines, $\lambda$ is the density of orders, $\mu$ is the density of servicing. The initial conditions are

$P_0(0) = 0, \quad P_k(0) = 0$ if at initial time $t = 0$ all lines are free.

Erlang system is reduced to the formula:

$$P_k(t) = \frac{\lambda^k / k!}{\sum_{k=0}^{n} \frac{\alpha^k}{k!}} \qquad k = 0,1\dots\dots\ n \tag{14}$$

where $\alpha = \lambda / \mu$ is known as a parameter for utility. The second case of Erlang system is described by elements which do not leave the system but they are waiting for a free line. The modified probabilities are:

$$P_k(t) = \frac{\alpha^k / k!}{\sum_{k=0}^{n} \frac{\alpha^k}{k!} + \frac{\alpha^{n+1}}{n.n!} \frac{1-(\frac{\alpha}{n})^m}{1-\frac{\alpha}{n}}} \qquad k = 0,1,.......n < m$$

$$P_{n+s}(t) = \frac{\alpha^n / n! * (\alpha / n)^s}{\sum_{k=0}^{n} \frac{\alpha^k}{k!} + \frac{\alpha^{n+1}}{n.n!} \frac{1-(\frac{\alpha}{n})^m}{1-\frac{\alpha}{n}}} \qquad s = 1,2,.......m \tag{15}$$

where $m$ is a number of lines for servicing. If $m$ is an infinite series $m \to \infty$ and $\alpha / n < 1$ the limit probability is

$$P_k(t) = \frac{\alpha^k / k!}{\sum_{k=0}^{n} \frac{\alpha^k}{k!} + \frac{\alpha^{n+1}}{n.n!} \frac{1}{1-\frac{\alpha}{n}}} \qquad k = 0,1,........n$$

$$P_{n+s}(t) = \frac{\alpha^n / n! * (\alpha / n)^s}{\sum_{k=0}^{n} \frac{\alpha^k}{k!} + \frac{\alpha^{n+1}}{n.n!} \frac{1}{1-\frac{\alpha}{n}}} \qquad s = 1,2,.........\infty$$

### 3.2 NON POISSON PROSSESES

These processes are described by density probabilistic functions and after that the functions are compared with appropriate test as chi-square or Kolmogorov-Smirnov test in order to select the best curve. In this paper two functions are used. The first one is gamma distribution defined as

$$f(x) = \frac{\lambda^r \, x^{r-1} \, e^{-\lambda x}}{\Gamma(r)}, \quad \lambda > 0 \quad r > 0 \quad x > 0 \tag{16}$$

where $\lambda$, $r$ are parameters and if $r$ is an integer the random variable has an Erlang distribution; $\Gamma(r)$ is a gamma function defined as

$$\Gamma(r) = \int_0^\infty x^{r-1} e^{-x} dx \tag{17}$$

The second function is known as the Weibull distribution which is defined by the formula

$$f(x) = \frac{\beta}{\delta} (\frac{x}{\delta})^{\beta-1} \exp\left[ -\left(\frac{x}{\delta}\right)^\beta \right], \quad \beta > 0 \quad \delta > 0 \quad x > 0 \tag{18}$$

where $\beta, \delta$ are parameters depending on gamma function, mathematical expectation $\mu$ and variance $\sigma^2$

$$\mu = \delta \, \Gamma(1 + \frac{1}{\beta}), \quad \sigma^2 = \delta^2 \, \Gamma(1 + \frac{2}{\beta}) - \delta^2 \left[ \Gamma\left(1 + \frac{1}{\beta}\right) \right]^2 \tag{19}$$

Chi-square method known as $\chi^2$ test determines the best parameter value for a given type of distributions that belongs to a sample of classification $K_j \quad j = 1.....k$ :

$$\chi^2 \equiv \left\{ \sum_j^k \frac{\tilde{f} - f(X_j, a)}{f(X_j, a)} \right\} \tag{20}$$

where $X_j$ denotes the midpoint of the interval of the $j-th$ class

The $\chi^2$ function measures the deviation of the relative frequency distribution $\tilde{f}_i$ from the best distribution $f(x,a)$ with a given parameter value $a$. This test is suitable for discrete and continuous random variables. Kolmogorov-Smirnov test of fit is very good for a small sample size but it is applicable only to continuous distributions. The maximum absolute difference is calculated by the formula

$$D_n = \sup_x \left| F(x) - F_n(x) \right|$$

where $F_n(x)$ is the empirical distribution function and $F(x)$ is a hypothetic distribution with a given significant level $\alpha$ and hypothesis $H_0$.

The hypothetic function $H_0$ is rejected, if $D_n > D_{\alpha,n}$. The bounds $D_{\alpha,n}$ are determined by tables or analytically.

### 3.3 AGING SERVICE PROCESSES

These processes are based on the analysis of a failure data rate assuming that this rate is a continuous function of the operating time (Ansel, J.I. and Philips, M.J. 1994). Many possible stochastic models are received later if the observed period and the failure data are available. The Crow's model applies a power function to describe the failure rate:

$$v(t) = \lambda \beta t^{\beta-1} \quad \lambda > 0 \quad \beta > 0 \tag{21}$$

where $v(t)$ is a time dependent failure rate, $\lambda$ is a scale parameter, $\beta$ is a growth parameter for improvement or deterioration in time; for $\beta > 1$ the failure rate increases and failures occur more often; on the contrary for $\beta < 1$ the failure rate decreases and the system improves. For $\beta = 1$ the failure rate is unchangeable.

According to Ansel and Philips the quality of the Crow's model can be checked by the means of confidence intervals using the next formula

$$\hat{\lambda} t_i^{\hat{\beta}} \pm z_{\alpha/2} \sqrt{\hat{\alpha}\, t_i^{\hat{\beta}}} \tag{22}$$

where $\hat{\lambda} = \frac{n}{t^{\hat{\beta}}}$ is a maximum likelihood estimator (ML) for the scale parameter $\lambda$, $t_i$ is the time of the failure event $i$, $\hat{\beta}$ is the ML estimator of the growth parameter $\beta$, $z_{\alpha/2}$ is $100(1-\alpha/2)$ i.e. the percent of the standard normal distribution.

Parameter $\hat{\beta}$ is estimated as

$$\hat{\beta} = \frac{n}{n \lg t - \sum_{i=1}^{n} \lg t_i} \tag{23}$$

where $n$ is the number of observations, $t$ is the end of the observation period and $t_i$ is the time of the $i-th$ failure.

A relatively wide range of data is needed so that the observations would be enclosed in 95% confidence interval. Moreover the model suffers from lack of sufficient information for repair and renewal of the components of the structure.

## 4 ANALYSIS OF THE DAM OPERATION AND NUMERICAL RESULTS

The analysis of the physical model according to the general scheme of the spillway (Fig.3) indicates two potential events of failure namely a high water level and water discharges – Fig.4 and Table 1.

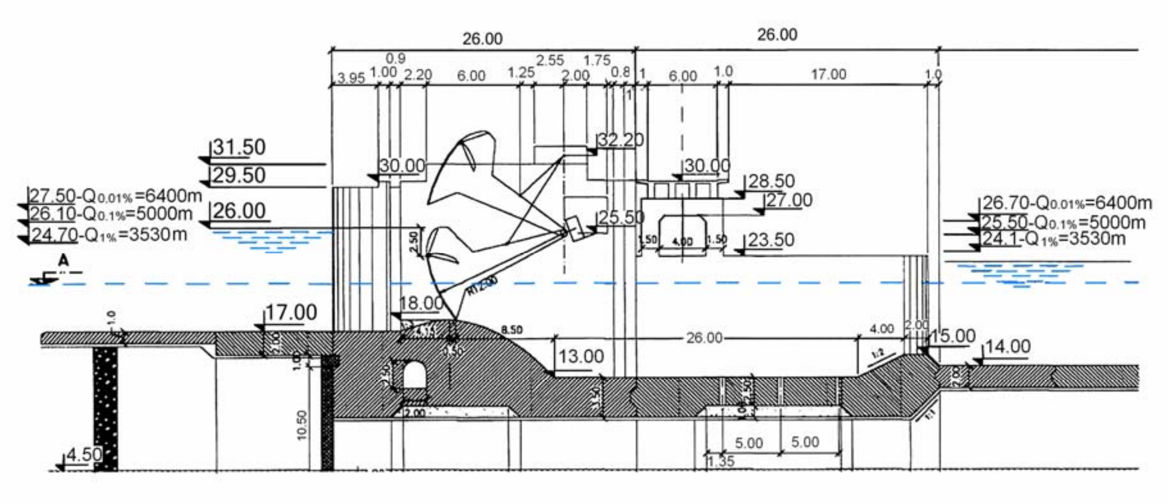


Fig.4 Water level during the drop and raising state of the gates

Table1
Upper and lower levels and discharges

| *Water discharges* $[m^3/s]$ | *Lower level* $[m]$ | *Levels in front of the gates* $[m]$ | | *Depth in front of the gates* $[m]$ | |
|---|---|---|---|---|---|
| | | *raising* | *drop* | *raising* | *drop* |
| $Q_{1\%} = 3530$ | 24.10 | 24.70 | 26 | 7.70 | 9 |
| $Q_{0.1\%} = 5000$ | 25.50 | 26.10 | 29.50 | 9.10 | 12.50 |
| $Q_{\max} = Q_{0.01\%} = 6400$ | 26.70 | 27.50 | 31.50 | 10.50 | 14.50 |

By the means of physical observations typical failure events of radial gates were investigated. According to the experimental studies the main characteristics of failure are due to vibrations, self-exiting vibrations, damages of the trunnion pin, operation chain, and electrical damages. First, the overall characteristics of damages are estimated through the Poisson formula (10), if the number of failures, frequencies and parameters is known. Second, $\chi^2$ test is used to determine the hypothesis of dependent failure.

Proposing the hypothetic Poisson function i.e. $H_0$ is a hypothesis of a given significance level $\alpha = 0.05$ (Harris, J.W. and Stocker H., 1998). The parameter $a$ in equation (10) is estimated as

$a = \frac{\sum_i m_i x_i}{100}$. The number of failures in the prototype for a period of 95 days is: $n = \sum_i m_i = 100$

The result is presented in the next table according to the Poisson formula (10) and the parameter is:

$$a = \frac{\sum_{i=0}^{5}(8.0 + 28.1 + 31.2 + 18.3 + 9.4 + 6.5)}{100} = 2.1;$$

Table 2
$\chi^2$ Test for $H_0$ hypothesis

| Number of failures $x_i$ | Frequency $m_i$ | Probability according to eq.(10) $P_i$ | $A=nP_i$ | $B=(m_i-nP_i)^2$ | B/A |
|---|---|---|---|---|---|
| 0 | 8 | 0.122 | 12.2 | 17.64 | 1.45 |
| 1 | 28 | 0.257 | 25.7 | 5.29 | 0.21 |
| 2 | 31 | 0.270 | 27 | 16 | 0.59 |
| 3 | 18 | 0.189 | 18.9 | 0.81 | 0.04 |
| 4 | 9 | 0.099 | 9.9 | 0.81 | 0.08 |
| 5 | 6 | 0.063 | 6.3 | 0.03 | 0.01 |
| - | n= $\sum_i m_i = 100$ | 1 | 100 | - | Sum=2.38= $\chi^2$ |

The degree of freedom is $\nu = x_i - n - 1 = 6 - 1 - 1 = 4$ where $x_i$ is the number of failures, $n$ is the number of parameters in the hypothetic function. The critical level is $\chi^2_{0,0.05,4} = 9.48$. The following inequality is implemented $\chi^2 = 2.35 < 9.48$. Therefore the hypothesis $H_0$ is accepted for further assessments.

The second example explains an order service process with failure and nine free lines, as the initial condition, shown in Fig. 3. This system has been modeling by the equation (14) and the linear system (13). The process essentially depends on the parameter of utility $\alpha$. Other parameters are $n = 9$, $\lambda = 10^{-1}\ [h^{-1}]$, $\mu = 12^{-1}\ [h^{-1}]$ and $\alpha = \frac{\lambda}{\mu} = 1.2$. The result is presented in the next table with selected parameters and the time in hours.

Table 3
Probability of the damaged lines and the average idle time

| N random selected free lines | Probability of served lines without failure | Average number of damaged lines | Probability of damaged lines | Average time for idle $[h]$ |
|---|---|---|---|---|
| 1 | 0.7757 | 0.931 | 0.1034 | ~ 104 |
| 2 | 0.5514 | 0.662 | 0.0735 | ~ 151 |
| 3 | 0.2068 | 0.422 | 0.0469 | ~ 243 |
| 4 | 0.1969 | 0.236 | 0.0262 | ~ 443 |
| 5 | 0.0935 | 0.112 | 0.0247 | ~ 473 |
| 6 | 0.0361 | 0.043 | 0.0048 | ~ 2482 |
| 7 | 0.0106 | 0.013 | 0.0014 | ~ 8474 |
| 8 | 0.0021 | 0.003 | 0.0003 | ~ 43.153 $10^3$ |
| 9 | 0.0020 | 0.002 | 0.0003 | ~ 44.045 $10^3$ |

By the means of an experimental study the time of failure was investigated for a bearing shaft of random selected gate (Fig. 4). The non Poisson distribution was used for probabilistic assessment on the bases of equations (16), (18). The Weibull distribution fits better for the time of observations than the gamma function. The mean time until the failure is determined by the mathematical expectation $\mu$. It is evaluated by the formulas given in the previous section as:

$\mu = 5000\,\Gamma\left[1+\frac{1}{0.5}\right] = 10.10^3\,[h]$, according to the equation (19) and the parameters are determined by experimental studies as $\beta = 1/2,\ \delta = 5.10^3$ [hours].

The cumulative function $F(x)$ gives the probability that a bearing shaft lasts more than $6.10^3$ [hours].The cumulative curve is given by the formulas of Weibull distribution $F(x) = e^{-(x/\delta)^\beta}$ and the result is:

$$P(x > 6000) = 1 - F(6000) = 1 - \exp-\left[(\frac{6000}{5000})^{1/2}\right] = 1 - e^{-1.095} = 1 - 0.337 = 0.663 \sim 66\%$$

Failure service processes described by the Crow's model needs a time dependent failure rate and observations provided in a prototype. Because of a small amount of data this model is far from completion and needs to be improved.

## 6 CONCLUSIONS

The analysis of the nine radial gates shows that the number of failures varies strongly and depends on the upper water level and on the rate of variation of this level. Related physical factors that affect to a great extend the potential events of failure are modeled by Reech-Froud criteria. By the means of these criteria the standard rating curve was plotted in appropriate scale. The results for upper, lower levels and water discharges are calculated for three typical discharges shown in Table 1. The upper level is related to the radial gates and their vibrations while the lower level mostly affects down stream section and energy dissipater. The scale coefficient for time allows the parameters in Poisson's and Erlang functions to be determined. Unfortunately the parameters in the Crow's model have not been received exactly in 95% confidence interval. As a result additional observations in a prototype are necessary. As a rule numerical results and physical models give reliable information source for different cases of potential events of failure.

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